

INTRODUCTION TO DIFFERENTIAL EQUATIONS



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Introduction to Differential Equations

1.1 Definitions and Terminology

1.2 Classification

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1.1 Definitions and Terminology

DEFINITION: differential equation

An equation containing the **derivative** of one or more **dependent variables**, with respect to one or more **independent variables** is said to be a **differential equation (DE)**.

(Zill, Definition 1.1, page 6).



1.2 Classification of D.E.

Differential Equations are classified by *types, order* and *linearity*.

There are two main *types* of differential equation: “ordinary” and “partial”.



Ordinary differential equation (ODE):

Differential equations that involve only **ONE** independent variable are called ordinary differential equations.

Examples:

$$\frac{dy}{dx} + 5y = e^x, \quad \frac{d^2y}{dx^2} - \frac{dy}{dx} + 6y = 0 \quad \text{and} \quad \frac{dx}{dt} + \frac{dy}{dt} = 2x + y$$

→ only *ordinary* (or *total*) derivatives

Partial differential equation (PDE)

Differential equations that involve **two or more** independent variables are called partial differential equations.

Examples:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} - 2 \frac{\partial u}{\partial t} \quad \text{and} \quad \frac{\partial u}{\partial y} = - \frac{\partial v}{\partial x}$$

→ only *partial* derivatives



1.3 Order of Differential equation

The *order* of a differential equation is the order of the highest derivative found in the DE.

$$\frac{d^2 y}{dx^2} + 5\left(\frac{dy}{dx}\right)^3 - 4y = e^x$$

second order

first order

Order of Differential equation



$$xy' - y^2 = e^x \quad \rightarrow \text{first order} \quad F(x, y, y') = 0$$

Written in differential form: $M(x, y)dx + N(x, y)dy = 0$

$$y'' = x^3 \quad \rightarrow \text{second order} \quad F(x, y, y', y'') = 0$$



1.4 Linear and Non linear D.E.

An n -th order differential equation is said to be **linear** if the function $F(x, y, y', \dots, y^{(n)}) = 0$ is linear in the variables $y, y', \dots, y^{(n-1)}$

$$\rightarrow a_n(x) \frac{d^n y}{dx^n} + a_{n-1}(x) \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_1(x) \frac{dy}{dx} + a_0(x) y = g(x)$$

→ there are no multiplications among **dependent variables** and **their derivatives**. All **coefficients** are functions of **independent variables**.

A **nonlinear** ODE is one that is *not linear*, i.e. does not have the above form.

Linear and Non linear differential equation

→ linear first-order ordinary differential equation

$$(y - x)dx + 4x dy = 0 \text{ or } 4x \frac{dy}{dx} + (y - x) = 0$$

→ linear second-order ordinary differential equation

$$y'' - 2y' + y = 0$$

→ linear third-order ordinary differential equation

$$\frac{d^3 y}{dx^3} + 3x \frac{dy}{dx} - 5y = e^x$$

Linear and Non linear differential equation

→ nonlinear first-order ordinary differential equation

$$(1 - y)y' + 2y = e^x \quad \text{coefficient depends on } y$$

→ nonlinear second-order ordinary differential equation

$$\frac{d^2 y}{dx^2} + \sin(y) = 0$$

nonlinear function of y

→ nonlinear fourth-order ordinary differential equation

$$\frac{d^4 y}{dx^4} + y^2 = 0$$

power not 1

1.5 Solution of ODE

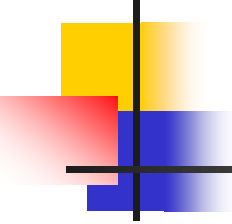


DEFINITION: Solution of an ODE

Any function ϕ , defined on an interval I and possessing at least n derivatives that are continuous on I , which when substituted into an n -th order ODE reduces the equation to an identity, is said to be a **solution** of the equation on the interval.

(Zill, Definition 1.1, page 8).

Solution of ODE



Namely, a solution of an n -th order ODE is a function which possesses at least n derivatives and for which

$$F(x, \phi(x), \phi'(x), \phi^{(n)}(x)) = 0 \quad \text{for all } x \text{ in } I$$

We say that ϕ *satisfies* the differential equation on I .

Solution of ODE

Verification of a solution by substitution

Example: $y'' - 2y' + y = 0$; $y = xe^x$

$$\rightarrow y' = xe^x + e^x, \quad y'' = xe^x + 2e^x$$

\rightarrow left hand side:

$$y'' - 2y' + y = (xe^x + 2e^x) - 2(xe^x + e^x) + xe^x = 0$$

= right-hand side



1.6 Types of Solutions

General Solution: Solutions obtained from integrating the differential equations are called general solutions. The general solution of a n th order ordinary differential equation contains n arbitrary constants resulting from integrating n times.

Particular Solution: Particular solutions are the solutions obtained by assigning specific values to the arbitrary constants in the general solutions.

Singular Solutions: Solutions that can not be expressed by the general solutions are called singular solutions.

Types of Solutions



Implicit solution

A relation $G(x,y)=0$ is said to be an **implicit solution** of an ODE on an interval I provided there exists at least one function ϕ that satisfies the relation as well as the **differential equation** on I .

→ a relation or expression $G(x,y)=0$ that defines a solution ϕ implicitly.

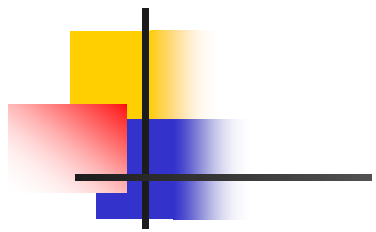
In contrast to an explicit solution $y=\phi(x)$



1.7 Conditions for Solution

Initial Condition: Constrains that are specified at the initial point, generally time point, are called initial conditions. Problems with specified initial conditions are called initial value problems.

Boundary Condition: Constrains that are specified at the boundary points, generally space points, are called boundary conditions. Problems with specified boundary conditions are called boundary value problems.



**Thank
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