INTRODUCTION TO DIFFERENTIAL EQUATIONS

Akhilesh Jain(Asst. Prof.)

Department of Mathematics

Corporate Institute of Science and Technology, Bhopal

akhiljain2929@gmail.com

Introduction to Differential Equations

1.1 Definitions and Terminology
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1.1 Definitions and Terminology

DEFINITION: differential equation

An equation containing the derivative of one or more dependent variables, with respect to one or more independent variables is said to be a **differential equation** (DE).

(Zill, Definition 1.1, page 6).

1.2 Classification of D.E.

Differential Equations are classified by ***types***, ***order*** and ***linearity***.

There are two main *types* of differential equation: “ordinary” and “partial”.

Classification of D.E.

Ordinary differential equation (ODE):
Differential equations that involve only **ONE** independent variable are called ordinary differential equations.

Examples:

$$\frac{dy}{dx} + 5y = e^x \quad , \quad \frac{d^2y}{dx^2} - \frac{dy}{dx} + 6y = 0 \quad \textbf{and} \quad \frac{dx}{dt} + \frac{dy}{dt} = 2x + y$$

➔**only *ordinary* (or *total*) derivatives**

Partial differential equation (PDE)
Differential equations that involve two or more independent variables are called partial differential equations.

Examples:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} - 2\frac{\partial u}{\partial t} \quad \textbf{and} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

➔**only *partial* derivatives**

1.3 Order of Differential equation

The *order* of a differential equation is the order of the highest derivative found in the DE.

$$\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^3 - 4y = e^x$$

second order **first order**

Order of Differential equation

$$xy' - y^2 = e^x$$ ➔ first order $F(x, y, y') = 0$

Written in differential form: $M(x, y)dx + N(x, y)dy = 0$

$$y'' = x^3$$ ➔ second order $F(x, y, y', y'') = 0$

1.4 Linear and Non linear D.E.

An *n*-th order differential equation is said to be **linear** if the function $F(x, y, y', \ldots\ldots y^{(n)}) = 0$ is linear in the variables $y, \quad y', \ldots \quad y^{(n-1)}$

$$\Rightarrow \quad a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + \ldots + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

➔there are no multiplications among **dependent variables** and **their derivatives**. All **coefficients** are functions of **independent variables**.

A **nonlinear** ODE is one that is *not linear*, i.e. does not have the above form.

Linear and Non linear differential equation

➔linear first-order ordinary differential equation

$$(y-x)dx+4xdy=0 \text{ or } 4x\frac{dy}{dx}+(y-x)=0$$

➔linear second-order ordinary differential equation

$$y''-2y'+y=0$$

➔ linear third-order ordinary differential equation

$$\frac{d^3y}{dx^3}+3x\frac{dy}{dx}-5y=e^x$$

Linear and Non linear differential equation

➔nonlinear first-order ordinary differential equation

$$(1-y)y' + 2y = e^x$$

coefficient depends on y

➔nonlinear second-order ordinary differential equation

$$\frac{d^2y}{dx^2} + \sin(y) = 0$$

nonlinear function of y

➔ nonlinear fourth-order ordinary differential equation

$$\frac{d^4y}{dx^4} + y^2 = 0$$

power not 1

1.5 Solution of ODE

DEFINITION: **Solution of an ODE**

Any function ϕ, defined on an interval I and possessing at least n derivatives that are continuous on I, which when substituted into an n-th order ODE reduces the equation to an identity, is said to be a **solution** of the equation on the interval.

(Zill, Definition 1.1, page 8).

Solution of ODE

Namely, a solution of an n-th order ODE is a function which possesses at least n derivatives and for which

$$F(x,\ \phi(x),\ \phi'(x),\ \phi^{(n)}(x)) = 0 \quad \text{for all } x \text{ in } I$$

We say that ϕ *satisfies* the differential equation on I.

Solution of ODE

Verification of a solution by substitution

Example: $y'' - 2y' + y = 0 \quad ; y = xe^x$

➔ $$y' = xe^x + e^x, \quad y'' = xe^x + 2e^x$$

➔left hand side:

$$y'' - 2y' + y = (xe^x + 2e^x) - 2(xe^x + e^x) + xe^x = 0$$

= right-hand side

1.6 Types of Solutions

General Solution: Solutions obtained from integrating the differential equations are called general solutions. The general solution of a nth order ordinary differential equation contains n arbitrary constants resulting from integrating times.

Particular Solution: Particular solutions are the solutions obtained by assigning specific values to the arbitrary constants in the general solutions.

Singular Solutions: Solutions that can not be expressed by the general solutions are called singular solutions.

Types of Solutions

Implicit solution

A relation $G\,(x,y)=0$ is said to be an

implicit solution of an ODE on an interval I provided there exists at least one function ϕ that satisfies the relation as well as the **differential equation** on I.

➔a relation or expression $G\,(x,y)=0$ that defines a solution ϕ implicitly.

In contrast to an explicit solution $y=\phi(x)$

1.7 Conditions for Solution

Initial Condition: Constrains that are specified at the initial point, generally time point, are called initial conditions. Problems with specified initial conditions are called initial value problems.

Boundary Condition: Constrains that are specified at the boundary points, generally space points, are called boundary conditions. Problems with specified boundary conditions are called boundary value problems.


Thank
You!!!