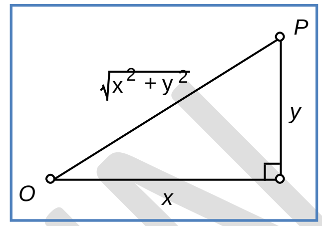
Coordinate Geometry

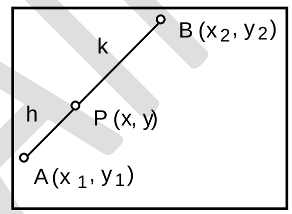
1. **Distance Formula :** Given two points $A(x_1, y_1)$, $B(x_2, y_2)$ in the rectangular coordinate plane. The distance between A and B is defined to be

$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} .$$



2. The distance from a point $P(x, y)$ to the origin is $\sqrt{x^2 + y^2}$.

3. **Section Formula:** Suppose the point $P(x, y)$ divides the segment joining $A(x_1, y_1)$, $B(x_2, y_2)$ **internally** in the ratio $AP : PB = h : k$, then $x = \dfrac{mx_2 + nx_1}{m+n}$ and $y = \dfrac{my_2 + ny_1}{m+n}$



Divides Externally: $x = \dfrac{mx_2 - nx_1}{m-n}$ and $y = \dfrac{my_2 - ny_1}{m-n}$

4. **Midpoint** : The midpoint of AB is $(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2})$.

5. Sometimes it is more convenient to denote $\lambda = \dfrac{AP}{AB}$ and the formula written as

$$\begin{cases} x = (1-\lambda)x_1 + \lambda x_2 \\ y = (1-\lambda)y_1 + \lambda y_2 \end{cases}$$

6. **Area of a Triangle:** If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle then

The Area of the triangle $\Delta = \dfrac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

These points are called collinear if OR $\Delta = 0$

7. **Centroid :** If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle then

Centroid= $(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3})$

8. **In centre:** If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle then

In centre = $(\dfrac{ax_1 + bx_2 + cx_3}{a+b+c}, \dfrac{ay_1 + by_2 + cy_3}{a+b+c})$, where *a, b,c* are the intercepts with axis.

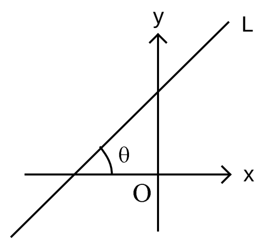
9. **Slope:** Given two distinct points $P(x_1, y_1)$, $Q(x_2, y_2)$. The slope of the segment/ray/line PQ is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

When $x_1 = x_2$, the slope is ∞. Some authors said the slope is *undefined* in this case.
Line with slope 0 is called *horizontal line*. Line with slope ∞ is called *vertical line*.

10. **Linear Equations in Two Variables (Equation of straight line)**
A linear equation in two variables is an equation which may be written in the form y = mx + b where m, and b are real numbers.

11. A straight line which makes a positive angle θ with x-axis has slope $m = \tan\theta$.



If $0 < \theta < 90°$, then $\tan\theta > 0$ and the line has positive slope. If $90° < \theta < 180°$, then the line has negative slope.

12. Equation of Straight Line.

(a) Point-slope Form: Given a point (x_1, y_1) on the line and slope *m*, the equation is $y - y_1 = m(x - x_1)$.

(b) Two-point Form: Given two points (x_1, y_1), (x_2, y_2) on the line, the equation is

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}.$$

(c) Intercept Form: Given *x*-intercept *a* and *y*-intercept *b*, the equation is $\frac{x}{a} + \frac{y}{b} = 1$.

(d) Slope-intercept Form : Given slope *m* and *y*-intercept *c*, the equation is $y = mx + c$.

(e) General Form: The equation of *any* straight line can be written as $Ax + By + C = 0$.

In this case, the slope and *y*-intercept are given by $\text{slope} = -\frac{A}{B}$ and $y\text{-intercept} = -\frac{C}{B}$.

When $A = 0$ and $B \neq 0$, the line is horizontal. When $A \neq 0$ and $B = 0$, the line is vertical. When $C = 0$, the line passes through origi**n.**

(f) Perpendicular form: $x\cos\alpha + y\sin\alpha = p$, where $p > 0$ *and* $0 \leq \alpha \leq 2\pi$

(g) Parametric Form: $x = x_1 + r\cos\alpha$, $y = y_1 + r\sin\alpha$

(h) The Equation of a Vertical Line has the form x = a, where a is the x-intercept of the vertical line.

(i) Perpendicular Lines: Two non-vertical lines are perpendicular if and only if their slopes are negative reciprocals of each other. The statement that they are negative reciprocals of each other may be stated algebraically with any one of the following equations.

$$m_1 = \frac{-1}{m_2} \quad \text{or} \quad m_2 = \frac{-1}{m_1} \quad \text{or} \quad m_1 m_2 = -1$$

(j) Parallel Lines: Two non-vertical lines are parallel if and only if they have different y-intercepts and they have the same slopes. i.e. $m_1 = m_2$

(k) Angle Between two Lines: If two lines are $a_1x + b_1y + c = 0$ and $a_2x + b_2y + c = 0$ then angle between them

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \quad \text{or} \quad \tan\theta = \left|\frac{a_2 b_1 - a_1 b_2}{a_1 a_2 + b_1 b_2}\right|$$

(l) Three lines are concurrent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

(m) Distance of a line $ax + by + c = 0$ from the point (x_1, y_1) is $p = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$

(n) Distance between two parallel lines: $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is $p = \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$

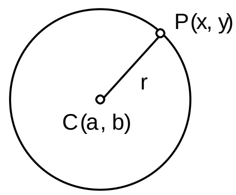
CONIC SECTION :

General equation of conic : $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$,

Circle (A=C, B=0) or ellipse ($A \neq C$ or $B \neq 0$)

(i) $B^2 - 4AC = 0$: Parabola (ii) $B^2 - 4AC > 0$: Hyperbola (iii) $B^2 - 4AC < 0$:

CIRCLE : A circle is a set of points for which they are equidistant from center.



Equations of Circle:

(a) Standard Form: The equation of circle centered at $C(a,b)$ with radius r is $(x-a)^2 + (y-b)^2 = r^2$.

(b) In particular, when $C = (0,0)$, the equation of circle centered at origin with radius r is $x^2 + y^2 = r^2$.

(c) General Form: The equation of *any* circle can be written as $x^2 + y^2 + Dx + Ey + F = 0$.

In this case, center $= (-\frac{D}{2}, -\frac{E}{2})$ and radius $= \frac{1}{2}\sqrt{D^2 + E^2 - 4F}$.

(d) Given the values of D, E and F, the equation $x^2 + y^2 + Dx + Ey + F = 0$ may NOT represent a circle. It represents a circle if and only if $D^2 + E^2 - 4F > 0$.

(e) If $D^2 + E^2 - 4F = 0$, we say the equation $x^2 + y^2 + Dx + Ey + F = 0$ represents a *point circle* .

(f) If $D^2 + E^2 - 4F < 0$, it represents an *imaginary circle* .

(g) A line for which intersects a circle C at exactly 1 point is called a *tangent* to the circle C at that point.

Intersection of Line and Circle:

Given a line $L: y = mx + c$ and a circle $C: x^2 + y^2 + Dx + Ey + F = 0$. To find their intersection, we consider

the system $\begin{cases} y = mx + c \\ x^2 + y^2 + Dx + Ey + F = 0 \end{cases}$

Substitute the first equation into the second one, we obtain $x^2 + (mx+c)^2 + Dx + E(mx+c) + F = 0$.

After simplification, it reduces to a quadratic equation $x^2 + px + q = 0$ --------------------------------- (1)

The number of intersections of L and C is equal to the number of (real) roots of (1). So,

$p^2 - 4q > 0 \Leftrightarrow$ 2 intersections	$p^2 - 4q = 0 \Leftrightarrow$ 1 intersections	$p^2 - 4q < 0 \Leftrightarrow$ no intersections

(h) Equation of the circle with diameter PQ, where $P(x_1, y_1)$ and $Q(x_2, y_2)$, is given by

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0.$$

Or

$$x^2 + y^2 - (x_1 + x_2)x - (y_1 + y_2)y + x_1x_2 + y_1y_2 = 0$$

The center is $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$, which is the midpoint of PQ. Hence, PQ is the diameter of this circle.

Summary:

Conic Section	Standard Form	Other Info.
Circle Centre (h,k) Radius r	$(x-h)^2+(y-k)^2=r^2$	Derived from the distance formula.
Parabola - Vertex (h,k) Focus $(h,k+a)$ Directrix at $y = k-a$	$(x-h)^2=4a(y-k)$	$a>0$ opens up, $a<0$ opens down
Parabola - Vertex (h,k) Foci $(h+a,k)$ Directrix at $x = h-a$	$(y-k)^2=4a(x-h)$	$a>0$ opens right, $a<0$ opens left
• **Ellipse** - Centre (h,k) • Horizontal major axis: a > b Vertices: $(h\pm a,k)$ Foci: $(h\pm c,k)$ • Vertical major axis: a > b Vertices: $(h,k\pm a)$ Foci: $(h,k\pm c)$ Eccentricity: $e = c/a$	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$	• The longer axis is called the major axis, the shorter axis is called the minor axis. • 'a' is the distance from the centre to each vertex (the end of the major axis). • 'b' is the distance from the centre to the end of the minor axis. • 'c' is the distance from the centre to each focus. $c^2=a^2-b^2$ • Length of major axis = 2a • Length of minor axis = 2b
• **Hyperbola -** Centre (h,k) • Horizontal transverse axis (x coefficient is positive) Vertices: $(h\pm a,k)$ Foci: $(h\pm c,k)$ Asymptote: $y-k=\pm\frac{b}{a}(x-h)$ • Vertical transverse axis (y coefficient is positive) Vertices: $(h,k\pm a)$ Foci: $(h,k\pm c)$ Asymptote : $y-k=\pm\frac{a}{b}(x-h)$ Eccentricity: $e = c/a$	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$	• 'a' is the distance from the centre to each vertex. • 'b' is a point on the conjugate axis but is not a point on the hyperbola (it helps determine asymptotes) • 'c' is the distance from the centre to each focus. $c^2=a^2+b^2$ • N.B. The transverse axis is not necessarily the longer axis but is associated with whichever variable is positive.