
SECOND ORDER LINEAR DIFFERENTIAL EQUATION WITH VARIABLE COEFFICIENTS

EQUATION WHOSE ONE SOLUTION IS KNOWN

Q 1. Solve $x\frac{d^2y}{dx^2}-(2x-1)\frac{dy}{dx}+(x-1)y=e^x$ given that $y = e^x$ is an integral.

[RGPV. Dec.2007, Feb.2010, June 2010]

Q 2. Solve $x^2\frac{d^2y}{dx^2}-2x(1+x)\frac{dy}{dx}+2(1+x)y=x^3$

Q 3. Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$ given that $x+\frac{1}{x}$ is one integral **[RGPV. Dec.2002,Jan .2006, June 2011]**

Q 4. Solve $\sin^2 x\frac{d^2y}{dx^2}=2y$, given that $y=\cot x$ is a solution. **[RGPV. Jan. 2007]**

Q 5. Solve $x\frac{d^2y}{dx^2}-(2x-1)\frac{dy}{dx}+(x-1)y=e^x$ **[RGPV. Dec.2003, 2007, Dec. 2008]**

Q 6. Solve $\frac{d^2y}{dx^2}-\cot x\frac{dy}{dx}-(1-\cot x)5y=e^x\sin x$ **[RGPV. Sept. 2009]**

Q 7. Solve $x\frac{d^2y}{dx^2}-\frac{dy}{dx}+(1-x)y=x^2e^{-x}$ given that $y=e^x$ is an integral.

NORMAL FORM (REMOVAL OF FIRST DERIVATIVE)

Q 8. Solve $x^2\frac{d^2y}{dx^2}-2(x^2+x)\frac{dy}{dx}+(x^2+2x+2)y=0$

Q 9. Solve $\frac{d^2y}{dx^2}-2\tan x\frac{dy}{dx}+5y=e^x\sec x$ **[RGPV. June. 2010]**

Q 10. Solve $\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+(4x^2-1)y=-3e^{x^2}\sin 2x$

METHOD OF CHANGING THE INDEPENDENT VARIABLE

Q 11. Solve $\frac{d^2y}{dx^2}-(1+4e^x)\frac{dy}{dx}+3e^{2x}y=2e^{2(x+e^x)}$ by using the method of changing the independent variable.

[RGPV. Jan. 2006]

Q 12. Solve $\cos x\frac{d^2y}{dx^2}+\sin x\frac{dy}{dx}-(2\cos^3 x)y=2\cos^5 x$ **[RGPV. June. 2003, Dec.2006]**

Q 13. Solve $(1+x^2)^2\frac{d^2y}{dx^2}+2x(1+x^2)\frac{dy}{dx}+4y=0$ **[RGPV. June. 2012]**

METHOD OF VARIATION OF PARAMETERS

Q 14. Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2}+y=\tan x$

Q 15. Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2}+a^2y=\sec ax$ **[RGPV. June. 2012]**

Q 16. Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2}+y=\sec x$ **[RGPV. Jan. 2007]**

Q 17. Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2}+a^2y=\cos ecax$

Q 18. Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2}+y=\cos ecx$

[RGPV. Dec. 2010, June. 2011]

Q 19. Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2}-y=\frac{2}{1+e^x}$

Q 20. Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=e^x\tan x$ **[RGPV. Jan. 2002]**

Q 21. Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2}+4y=4\tan 2x$

[RGPV. Dec. 2004, 2007, June. 2008, Feb. 2010]

Q 22. Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+6y=\frac{e^{3x}}{x^2}$ **[RGPV. Jan. 2006]**

Q 23. Apply the method of variation of parameters to solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x^2e^x$ **[RGPV. Sept. 2009]**

Q 24. Solve $x^2\frac{d^2y}{dx^2}-2x(1+x)\frac{dy}{dx}+2(1+x)y=x^3$ **[RGPV. June 2007]**

SOLUTION BY SERIES METHOD: FOR ORDINARY POINT

Q 25. Solve the equation using by series method $\frac{d^2y}{dx^2}+xy=0$ **[RGPV. June 2012]**

Q 26. Solve the equation using by series method $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+4y=0$ **[RGPV. June 2008]**

Q 27. Solve the equation using by series method $(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$ **[RGPV. June 2002, Dec.2003]**

Q 28. Solve the equation using by series method $(2-x^2)\frac{d^2y}{dx^2}+2x\frac{dy}{dx}-2y=0$

Q 29. Solve the equation using by series method

$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0$ (Legendre's differential equation) **[RGPV. June 2006]**

FOR REGULAR SINGULARITY (FROBENIUS METHOD):

Q 30. Solve the equation using by series method $x^2\frac{d^2y}{dx^2}+(2x^2-x)\frac{dy}{dx}+y=0$ **[RGPV. June 2004]**

Q 31. Solve the equation using by series method $9x(1-x)\frac{d^2y}{dx^2}-12\frac{dy}{dx}+4y=0$

Q 32. Solve the equation using by series method $2x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+(1-x^2)y=0$

Q 33. Solve the equation using by series method $(2x+x^3)\frac{d^2y}{dx^2}-\frac{dy}{dx}-6xy=0$ **[RGPV. Dec. 2004]**

Q 34. Solve the equation using by series method $2x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-(x+1)y=0$

Q 35. Solve the equation using by series method

$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-n^2)y=0$ (Bessel's differential equation of order n)

Q 36. Solve the equation using by series method **[RGPV. Dec. 2008, Sep.2009,June 2011,]**

$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-4)y=0$ (Bessel's differential equation of order 2)

Q. 37. Solve the equation using by series method

$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-1)y=0$ (Bessel's differential equation of order 1)

Q. 38. Solve the equation using by series method

$x\frac{d^2y}{dx^2}+\frac{dy}{dx}+xy=0$ (Bessel's differential equation of order 0)