CORPORATE
GROUP OF INSTITUTES, BHOPAL
IMPORTANT QUESTIONS
UNIT-2 : INTEGRAL CALCULUS

Sem-3 , Sub: Engg. Mathematics -II, BE-301 , Name of the faculty: Akhilesh jain

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CHAPTER : LAPLACE TRANSFORMS

Q .1. Find the Laplace transforms of (1) $L\ \sin\sqrt{t}$ (2) $L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\}$

Q .2. Find the Laplace transforms of $f(x)=\begin{cases} t\ ;\ 0<t<4 \\ 5\ ;t>4 \end{cases}$

Q .3. Find the Laplace transforms of $f(x)=\begin{cases} \sin t\ ;\ 0<t<\pi \\ 0\ \ ;\ t>\pi \end{cases}$

Q .4. Find the Laplace transforms of (1) $L\ e^{-4t}\sin 3t$ **[RGPV. Dec.2007]**

(2) $L\ e^{-t}(3\sinh 2t-5\cosh 2t)$ (3) $L\ e^{-3t}t^4$ (4) $L\ e^{-3t}(3\sin 2t+4\cos 2t)$

Q .5. Find the Laplace transforms of (1) $L\ t\sin at$ (2) $L\ t^2\sin at$ (3) $L\ t\ \cos at$

(4) $L\ t^2\cos at$ (5) $L\ t^2\sin at$ **[RGPV. Dec. 2004 , 2010, 2011]**

Q .6. Find the Laplace transforms of $L\left\{\frac{\sin t}{t}\right\}$

Q .7. Find $L\left\{\frac{1-\cos 2t}{t}\right\}$ ***[RGPV Dec. 2003, June 2007, 2012]***

Q .8. Show that the $L\left\{\frac{\cos at}{t}\right\}$ does not exist , but $L\left\{\frac{\cos at-\cos bt}{t}\right\}$ exist and find it.

[RGPV. Feb. 2010, Dec. 2010, June 2012]

Q .9. Find $L\left\{\frac{1-e^t}{t}\right\}$ **[RGPV. Dec. 2011]**

Q .10. Find the Laplace transforms of (1) $L\left\{\frac{e^{-t}\sin t}{t}\right\}$ ***[RGPV. June 2002]***

(2) $L\left\{\int_0^t\frac{\sin t}{t}dt\right\}$ ***[RGPV. Dec. 2003]*** (3) $L\left\{\int_0^t\frac{e^t\sin t}{t}dt\right\}$ ***[RGPV. Sep. 2009]***

Q .11. Find the Laplace transforms of (1) $L\ te^{-2t}\sin 2t$ (2) $L\ te^{-4t}\sin 3t$ (3) $L\ te^{4t}\cos 3t$

Q .12. Find the value of (1) $\int_0^{\infty} \frac{e^{-t}\sin t}{t}dt$ (2) $\int_0^{t} \frac{e^{t}\sin t}{t}dt$

Q .13. Let $F(t) = f(p)$ be a periodic function with period T, then prove that $L\{F(t)\} = \frac{\int_0^T e^{-pt}F(t)dt}{1-e^{-pT}}$ [***RGPV Jan. 2007,Dec. 2011]***

Q .14. If L $\{F(t)\} = f(p)$ then prove that $L\{tF(t)\} = -f'(p)$ ***[RGPV. Dec. 2001]***

Q .15. Using Laplace transform Prove that (1) $\int_0^{\infty} \frac{\sin t}{t}dt = \frac{\pi}{2}$ (2) $\int_0^{\infty} \frac{e^{-at}-e^{-bt}}{t}dt = \log\frac{b}{a}$

(3) $\int_0^{\infty} t\,e^{-3t}\sin t\,dt = \frac{3}{50}$ (4) $\int_0^{\infty} t\,e^{-2t}\cos t\,dt = \frac{3}{25}$

TOPIC : INVERSE LAPLACE TRANSFORMS

Q .16. Find the Inverse Laplace Transform of $L^{-1}\left\{\frac{6}{2p-3} - \frac{3+4p}{9p^2-16} + \frac{8-6p}{16p^2+9}\right\}$

Q .17. Find the Inverse Laplace Transform of (1) $L^{-1}\left\{\log\frac{p(p+1)}{(p^2+4)}\right\}$ ***[RGPV. Dec. 2004]***

(2) $L^{-1}\left\{\log\frac{p+1}{p-1}\right\}$ [***RGPV. June 2005, Feb. 2010]***

(3) $L^{-1}\left\{\log\frac{p^2-1}{p^2}\right\}$ ***[RGPV. Jan 2006]*** (4) $L^{-1}\left\{\log(1+\frac{1}{p})\right\}$

Q .18. Find the Inverse Laplace Transform of (1) $L^{-1}\left\{\frac{1}{p^2-5p+6}\right\}$

(2) $L^{-1}\left\{\frac{2p^2-6p+5}{(p-1)(p-2)(p-3)}\right\}$ (3) $L^{-1}\left\{\frac{p+1}{(p+2)^2}\right\}$

(4) $L^{-1}\left\{\frac{3}{(p+1)^2}\right\}$ (5) $L^{-1}\left\{\frac{p}{(p^2+1)(p^2+4)}\right\}$ (6) $L^{-1}\left\{\frac{p^2}{(p^2+a^2)(p^2+b^2)}\right\}$

Q .19. Use convolution theorem to evaluate. (1) $L^{-1}\left\{\frac{1}{(p-2)(p-3)}\right\}$ (2) $L^{-1}\left\{\frac{1}{(p^2+a^2)^2}\right\}$

(3) $L^{-1}\left\{\frac{p}{(p^2+a^2)^2}\right\}$ [***RGPV. June 2011, 2012, Dec, 2011]***

(4) $L^{-1}\left\{\frac{p^2}{(p^2+a^2)(p^2+b^2)}\right\}$ ***[RGPV. June 2006, 2008,Dec. 2008,2010]***

(5) $L^{-1}\left\{\frac{1}{(p+3)(p^2+9)}\right\}$ ***[RGPV. June 2011]***

Q .16. Evaluate $L^{-1}\left\{\frac{p^2+6}{(p^2+1)(p^2+4)}\right\}$ ***[RGPV. June 2002, 2012 Jan. 2006]***

Q .17. Evaluate $L^{-1}\left\{\frac{p+2}{(p^2+4p+5)^2}\right\}$ ***[RGPV. Sep. 2009]***

SOLUTION OF DIFFERENTIAL EQUATIONS BY LAPLACE TRANSFORMS

Q .20. Using Laplace Transformation solve the following differential equation

$$\frac{d^2y}{dt^2}+9y=6\cos 3t \qquad y(0)=2,\ y'(0)=0,$$

Q .21. Using Laplace Transformation solve the following differential equation $\frac{d^2x}{dt^2}+9x=\cos 2t$,if

$x(0)=1,\ x\left(\frac{\pi}{2}\right)=-1$ ***[RGPV .June. 2008]***

Q .22. Using Laplace Transformation solve the following differential equation

$y''-2y'+y=e^t$, $y(0)=2$, $y'(0)=-1$ ***[RGPV. Dec. 2008 , 2011 , Feb. 2010]***

Q .23. Using Laplace Transformation solve the following differential equation

$\frac{d^3y}{dt^3}-3\frac{d^2y}{dt^2}+3\frac{dy}{dt}-y=t^2e^t$ Where $y(0)=1,\ y'(0)=0\ ,\ y''(0)=-2$ ***[RGPV. Dec. 2007]***

Q .24. Using Laplace Transformation solve the following differential equation $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y=\sin t$

Where $y(0)=1\ ,\ y'(0)=0\ ,$ ***[RGPV. Dec. 2007]***

Q .25. Using Laplace Transformation solve the following differential equation

$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+5y=3e^{-x}\sin x$ Where $y(0)=0, y'(0)=1,$ ***[RGPV. June. 2007]***

Q .26. Using Laplace Transformation solve the following differential equation $\frac{d^3y}{dx^3}+2\frac{d^2y}{d^2x}-\frac{dy}{dx}-2y=0$

Where $y(0)=1,\ y'(0)=2,\ y''(0)=2$

Q .27. Solve the following simultaneous differential equation by Laplace transform

$3\frac{dx}{dt}-y=2t$, $\frac{dx}{dt}+\frac{dy}{dt}-y=0$ With the conditions $x(0)=y(0)=0$

Q .28. Solve the following simultaneous differential equation by Laplace transform $\frac{dx}{dt}+y=\sin t$,

$\frac{dy}{dt}+x=\cos t$ With the conditions $x(0)=2\ ,\ y(0)=0$

Q .29. Solve the following simultaneous differential equation by Laplace transform

Q .30. $\frac{dx}{dt}+5x-2y=t$, $\frac{dy}{dt}+2x+y=0$ With the conditions $x(0)=y(0)=0$ ***[RGPV. Sep. 2009]***