CORPORATE
GROUP OF INSTITUTES, BHOPAL
IMPORTANT QUESTIONS
UNIT-1 FOURIER SERIES & FOURIER TRANSFORM

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CHAPTER: FOURIER SERIES

Q. 1 . Find a Fourier series of function $f(x) = x^2$ from $x=-\pi$ to $x=\pi$.

Q. 2 . Find the fourier series to represent the function $x-x^2$ in the interval $-\pi<x<\pi$. Also deduce that $\frac{\pi^2}{12}=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+$............ ***[RGPV-June. 2006,2013 Feb. 2010]***

Q. 3 . Find a series of sines and cosines of multiples x which will represent $x+x^2$ in the interval $-\pi<x<\pi$. Hence show that $\frac{\pi^2}{6}=1+\frac{1}{2^2}+\frac{1}{3^2}+$............ ***[RGPV-Jan. 2006]***

Q. 4 . Find the Fourier series expansion for $f(x)=2x-x^2$ in (0,3) and hence deduce that

$\frac{\pi^2}{12}=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+$............ ***[RGPV-June 2008]***

Q. 5 . Find the Fourier series to represent *the* function $f(x)=|x|$ in the interval $-\pi<x<\pi$. And hence deduce $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+$............ ***[RGPV-Dec. 2005, 2007]***

Q. 6 . Find a Fourier series of the function $f(x)=|\cos x|$. from. $-\pi<x<\pi$

Q. 7 . Find a Fourier series of the function $f(x)=|\sin x|$. from. $-\pi<x<\pi$

Q. 8 . Expand *f(x)= x sinx , 0 < x < 2π in a Fourier Series.*

[RGPV- June 2004, 2007, Dec. 2008,, 2011]

Q. 9 . Expand $f(x)=\sqrt{1-\cos x}$ *, 0 < x < 2π in a Fourier Series. Hence evaluate*

$\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+$.......... ***[RGPV Sept. 2009]***

Q. 10 . Find a Fourier series of the function $f(x)=x^2 \quad -l<x<l$ ***[RGPV Jan 2005, Feb. 2010]***

Q. 11 . Find a Fourier series of the function $f(x)=x^2$ $, -\pi<x<\pi$. Using two values of *f(x)* show that

$\frac{\pi^4}{90}=\frac{1}{1^4}-\frac{1}{2^4}+\frac{1}{3^4}-\frac{1}{4^4}+$............ ***[RGPV June . 2008]***

[Hint : use Parseval's Identity for deduction]

CHAPTER : DISCONTINUOUS FUNCTION

Q. 12 . Find a Fourier series of the function $f(x)=\begin{cases}-\pi \ ; -\pi<x<0 \\ x \ ; 0<x<\pi\end{cases}$

Hence deduce $\dfrac{\pi^2}{8}=\dfrac{1}{1^2}+\dfrac{1}{3^2}+\dfrac{1}{5^2}+............$ ***[RGPV. Dec.2004, Jan.2007,Dec.2008]***

Q. 13 . Find a Fourier series of the function $f(x)=\begin{cases}kx \ ; 0<x<l/2 \\ k(l-x) \ ; l/2<x<l\end{cases}$ **[RGPV. Dec. 2004, Jan 2006]**

Q. 14 . Expand in a Fourier series the periodic function f(x) , with period 2l which on the interval (-l, l) is given by the equation $f(x)=|x|$, where $|x| = \begin{cases}-l \ , -l \le x \le 0 \\ l \ , 0 \le x \le l\end{cases}$ [***RGPV-Jan. 2001]***

Q. 15 . A periodic function of period 4 is defined as $f(x)=|x|$ *, -2< x < 2. Find its Fourier series expansion.* [***RGPV-Dec. 2002]***

Q. 16 . Expand the function *f(x)* in Fourier series in the interval $-\pi<x<\pi$*:*

$$f(x) = \begin{cases}x \ , -\pi \le x \le 0 \\ 2x \ , 0 \le x \le \pi\end{cases}$$

[***RGPV-Dec. 2001,June 2011]***

CHAPTER : HALF-RANGE FOURIER SERIES

Q. 17 . Find the half range sine Fourier series for the function *f(x) = x* in the interval $-\pi \le x \le \pi$
[RGPV *June 2006,2007]*

Q. 18 . Find the half range sine Fourier series for the function *f(x) = x* in the interval $0 \le x \le 2$
[RGPV *Jan 2007]*

Q. 19 . Develop $f(x)=\sin(\dfrac{\pi x}{l})$ in a half range cosine series in the range *0< x<l* ***[RGPV Dec. 2005]***

Q. 20 . Find the Fourier sine and cosine series of $f(x)=\begin{cases}x \ ; \ 0<x<\pi/2 \\ 0 \ ; \pi/2<x<\pi\end{cases}$

Q. 21 . Find the half range cosine Fourier series of $f(x) = \begin{cases}kx \ ; & 0<x<\frac{l}{2} \\ k(l-x); & \frac{l}{2}<x<l\end{cases}$

Q. 22 . Find the Fourier series in the sine terms of $f(x)=\begin{cases}\dfrac{1}{4}-x \ ; \ 0<x<1/2 \\ x-\dfrac{3}{4} \ ; 1/2<x<1\end{cases}$ **[RGPV. Sept. 2009]**

Q. 23 . Find the half range sine r series for the function $f(x)=\pi x-x^2$ *, 0< x < π,. And deduce that*

$$\sum_{n=1}^{\infty}\frac{1}{n^6}=\frac{\pi^6}{945}$$

[RGPV *June 2005]*

CHAPTER: FOURIER TRANSFORMS

Q. 24 . Find the Fourier complex transform of *f(x)* ,if $f(x) = \begin{cases} e^{iwx}, & a < x < b \\ 0 \;, & x < a \,, \; x > b \end{cases}$

Q. 25 . Find the Fourier transform of $f(x) = \begin{cases} 1 \,, & |x| < 1 \\ 0 \,, & |x| > 1 \end{cases}$

Hence evaluate (i) $\int_{-\infty}^{\infty} \frac{\sin sa \cos sx}{s} \, ds$ (ii) $\int_{0}^{\infty} \frac{\sin s}{s} \, ds$

Q. 26 . Find the Fourier transform of $f(x) = \begin{cases} 1-x^2 \,, & |x| \le 1 \\ 0 \,, & |x| > 1 \end{cases}$

Hence evaluate $\int_{0}^{\infty} \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2} \, dx$

Q. 27 . Show that the Fourier transforms of $f(x) = e^{-\frac{x^2}{2}}$ is self reciprocal.

Q. 28 . Find the Fourier transform of $f(x) = \begin{cases} x, & |x| \le a \\ 0, & |x| > a \end{cases}$

Q. 29 . Find the **cosine transform** of the function $f(x) = \begin{cases} \cos x, & 0 < x < a \\ 0, & x > a \end{cases}$

Q. 30 . Find the **sine transform** of the function , $f(x) = \begin{cases} \sin x, & 0 < x < a \\ 0, & x > a \end{cases}$

Q. 31 . Find the **sine transform** of the function , $f(x) = \frac{1}{x}$

Q. 32 . Find the **sine and cosine transform** of the function , $f(x) = e^{-ax}$

Q. 33 . Find the cosine transform of the function $f(x) = e^{-|x|}, x \ge 0$ and prove that $\int_{0}^{\infty} \frac{\cos sx}{1+s^2} \, ds = \frac{\pi}{2} e^{-x}$

Q. 34 . Find the sine transform of the function $f(x) = e^{-|x|}, x \ge 0$ and prove that

$$\int_{0}^{\infty} \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m}, m > 0$$

Q. 35 . Find the Fourier sine transform of $f(x) = e^{-\frac{ax}{x}}$. Hence find the Fourier sine transform of *1/x.*

Q. 36 . Find the Fourier sine transform of $f(x) = e^{-x^2}$. Hence find the Fourier sine transform of *1/x.*

Q. 37 . Find the Fourier transform of $f(x) = \begin{cases} x & ; 0 < x < 1 \\ 2-x & ; 1 < x < 2 \\ 0 & ; x > 2 \end{cases}$