Laplace Transform:

Function *F(t)*	Laplace Traqnsform $L\{F(t)\} = f(p)$	Function $L^{-1}\{f(p)\}$	Inverse Laplace Transform *F(t)*
$L\{F(t)\} = f(p)$	$\int_0^\infty e^{-pt} F(t)\,dt = f(p)$		
$L\{1\}$	$\frac{1}{p}$, p>0	$L^{-1}\{\frac{1}{p}\}$, p>0	1
$L\{t\}$	$\frac{1}{p^2}$, p>0	$L^{-1}\{\frac{1}{p^2}\}$, p>0	t
$L\{t^n\}$	$\frac{n!}{p^{n+1}}$, p>0	$L^{-1}\{\frac{1}{p^{n+1}}\}$, p>0	$\frac{t^n}{n!}$
$L\{e^{at}\}$	$\frac{1}{p-a}$, p>a	$L^{-1}\{\frac{1}{p-a}\}$, p>a	e^{at}
$L\{cosat\}$	$\frac{p}{p^2+a^2}$, p>0	$L^{-1}\{\frac{p}{p^2+a^2}\}$, p>0	cosat
$L\{sinat\}$	$\frac{a}{p^2+a^2}$, p>0	$L^{-1}\{\frac{1}{p^2+a^2}\}$, p>0	$\frac{sinat}{a}$
$L\{coshat\}$	$\frac{p}{p^2-a^2}$, |p|>0	$L^{-1}\{\frac{p}{p^2-a^2}\}$, |p|>0	coshat
$L\{sinhat\}$	$\frac{a}{p^2-a^2}$, |p|>0	$L^{-1}\{\frac{a}{p^2-a^2}\}$, |p|>0	$\frac{sinhat}{a}$
Linear Property $L\{aF_1(t)+bF_2(t)\} = aL\{F1(t)\}+bL\{F2(t)\}$		***Linear Property :*** $L^{-1}\{aF_1(t)+bF_2(t)\} = a\,L^{-1}F_1(t)+b\,L^{-1}F_2(t)$	
First Shifting (Translation)Theorem $L\{e^{at} F(t)\} = f(p-a)$, where $f(p)= L\{F(t)\}$		***First Shifting (Translation)Theorem*** $L^{-1}\{f(p-a)\} = e^{at} L^{-1} F(t)\}$ where $f(p)= L\{F(t)\}$	
Second Shifting *(Translation)* Theorem: If $G(t)=\begin{cases} F(t-a) & ; t>a \\ 0 & ; t<a \end{cases}$ Then $L\{G(t)\} = e^{-ap} f(p)$, where $f(p)= L\{F(t)\}$		**Second Shifting *(Translation)* Theorem** $L^{-1}\{e^{-ap} f(p)\}=G(t)$, where $f(p)= L\{F(t)\}$ and $G(t)=\begin{cases} F(t-a) & ; t>a \\ 0 & ; t<a \end{cases}\}$	
Change of Scale property $L\{F(at)\}=\frac{1}{a} f(\frac{p}{a})$, where $f(p)= L\{F(t)\}$		**Change of Scale property** $L^{-1}\{f(ap)\} = \frac{1}{a} F(\frac{t}{a})$ where $F(t)= L^{-1}\{f(p)\}$,	
Differentiation Theorem $L\{F'(t)\} = p\,L\{F(t)\} - F(0)$			
$L\{F^n(t)\}= p^n L\{F(t)\} - p^{n-1} F(0) - p^{n-2} F'(0) - \ldots\ldots\ldots \ldots. - F^{n-1}(0)$			
Integral Theorem If *F(t)* is piecewise continuous function and $\|F(t)\| \le Me^{at}$ then $L\{\int_0^t F(x)dx\} = \frac{1}{p} L\{F(t)\}$			

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Multiplication Theorem $L\{t\,F(t)\} = (-1)\frac{d}{dp}f(p) = -f'(p)$ $L\{t^n F(t)\} = (-1)^n \frac{d^n}{dp^n} f(p)$	***Multiplication Theorem*** $L^{-1}\{p\,f(p)\} = F'(t)$
Division Theorem $L\{\frac{F(t)}{t}\} = \int_p^\infty f(p)dp$	***Division Theorem*** $\{L^{-1}\{\frac{f(p)}{p}\} = F(t) = \int_0^t f(p)dp$
Laplace Transform of integrals: $L\{\int_0^t F(t)dt\} = \frac{1}{p} f(p)$	***Division Theorem (General form)*** $: L^{-1}\{\frac{f(p)}{p^n}\} = F(t) = \int_0^t \text{.............} \int_0^t f(p)dp^n$
Fundamental theorem of periodic function *If F(t) is a periodic function of period T then* $L\{F(t)\} = \frac{\int_0^T e^{-pt} F(t)dt}{1-e^{-pT}}$	

Convolution Theorem: If $L^{-1}\{f(p)\} = F(t)$ and $L^{-1}\{g(p)\} = G(t)$, where F and G are two function of Class A then

$$L^{-1}\{f(p).g(p)\} = \int_0^t F(x)G(t-x)dx = F*G$$

Heaviside's Expansion Theorem: If *f(p)* and *g(p)* are two polynomials in *p* ,where *degree f(p) <degree g(p)* . If g(p) is a polynomial of n- distinct zeros $\alpha_1, \alpha_2, \ldots\ldots\ldots\alpha_n$ then

$$L^{-1}\{\frac{f(p)}{g(p)}\} = \sum_{i=1}^{n} \frac{f(\alpha_i)}{g'(\alpha_i)} e^{\alpha_i t} = \frac{f(\alpha_1)}{g'(\alpha_1)} e^{\alpha_1 t} + \frac{f(\alpha_2)}{g'(\alpha_2)} e^{\alpha_2 t} + \ldots\ldots\ldots \frac{f(\alpha_i)}{g'(\alpha_i)} e^{\alpha_i t}$$

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Fourier series: The **Fourier series** of *f* (*x*) on the interval (–*L*, *L*) is

$$f\;\; x \;=\; \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right)$$

Where $a_n = \frac{1}{L}\int_{-L}^{L} f\;\; x \cos\left(\frac{n\pi x}{L}\right)dx, \quad n = 0, 1, 2, 3, \ldots$ & $b_n = \frac{1}{L}\int_{-L}^{L} f\;\; x \sin\left(\frac{n\pi x}{L}\right)dx, \quad n = 1, 2, 3, \ldots$

The {a_n, b_n} are the Fourier coefficients of *f* (*x*).

Half-Range Fourier Series

A Fourier series for *f* (*x*), valid on [0, *L*], may be constructed by extension of the domain to [–*L*, *L*].
An odd extension leads to a **Fourier sine series**:

$$f\;\; x = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right) \qquad \text{Where } b_n = \frac{2}{L}\int_0^L f\;\; x \sin\left(\frac{n\pi x}{L}\right)dx, \quad n = 1, 2, 3, \ldots$$

An even extension leads to a **Fourier cosine series**:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right),$$

Where $a_0 = \frac{2}{L}\int_0^L f(x)\,dx$, & $a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$, $n = 0, 1, 2, 3, \ldots$

and there is automatic continuity of the Fourier cosine series at $x = 0$ and at $x = \pm L$.

Convergence

At all points $x = x_o$ in $(-L, L)$ where $f(x)$ is continuous and is either differentiable or the limits $\lim_{x \to x_o^-} f'(x)$ and $\lim_{x \to x_o^+} f'(x)$ both exist, the Fourier series converges to $f(x)$.

At finite discontinuities, (where the limits $\lim_{x \to x_o^-} f'(x)$ and $\lim_{x \to x_o^+} f'(x)$ both exist), the Fourier series converges to $\frac{f(x_o-) + f(x_o+)}{2}$,

(Using the abbreviations $f(x_o-) = \lim_{x \to x_o^-} f(x)$ and $f(x_o+) = \lim_{x \to x_o^+} f(x)$).

Note: The cosine functions (and the function 1) are even, while the sine functions are odd.

If $f(x)$ is even ($f(-x) = +f(x)$ for all x), then $b_n = 0$ for all n, leaving a Fourier cosine series (and perhaps a constant term) only for $f(x)$.

If $f(x)$ is odd ($f(-x) = -f(x)$ for all x), then $a_n = 0$ for all n, leaving a Fourier sine series only for $f(x)$.

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