CORPORATE INSTITUTE OF SCIENCE AND TECHNOLOGY BHOPAL

IMPORTANT QUETIONS OF M-1

BRANCH – ME, CE, CSE

SESSION : FEB 2015-JUNE 2015

UNIT – 1: [DIFFERENTIAL CALCULUS]

Q.N.1: Expand $\tan\left(x+\frac{\pi}{4}\right)$ by Taylor's theorem and hence find the value of $\tan 46.5^o$ up to four significant digits. **Ans :** , $\tan 46.5^o$ =1.0537

Q.N.2: Expand $e^{a\sin^{-1}x}$ by Maclaurin's theorem and prove that

$$e^{\theta} = 1+\sin\theta+\frac{1}{\lfloor 2}\sin^2\theta+\frac{2}{\lfloor 3}\sin^3\theta+.....$$ Where $\theta=\sin^{-1}x$.

Q.N.3: If $u=\tan^{-1}\left[\frac{x^3-y^3}{x+y}\right]$, then prove that

(i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$ **(ii)** $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=2\cos 3u\sin u$

Q.N.4: (i) Examine the function x^3+y^3-3axy for maxima and minima. **Ans**: $F_{max}=a^3$.

(ii) Show that the radius of curvature at any point on the cardioids $r=a(1-\cos\theta)$ is $\frac{2}{3}\sqrt{2ar}$ .Also prove that $\frac{\rho^2}{r}$ is constant.

Q.N.5: (i) If $u=\log(x^3+y^3+z^3-3xyz)$, Show that:

(a) $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3}{x+y+z}$ **(b)** $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=\frac{-9}{(x+y+z)^2}$

(ii) If $x^x y^y z^z=c$, then show that $\frac{\partial^2 z}{\partial x\partial y}=-(x.\log e.x)^{-1}$ at $x=y=z$.

UNIT – 2 : (INTIGRAL CALCULAS)

Q.N.6: (i) Evaluate: $\lim_{x\to\infty}\left[\frac{(n+1)(n+2).......(n+n)}{n^n}\right]^{\frac{1}{n}}$ **Ans:** $\frac{4}{e}$

(ii) Evaluate : $\lim_{x\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\text{.........}\left(1+\frac{n^2}{n^2}\right)\right]^{\frac{1}{n}}$ **Ans:** $2e^{\frac{\pi}{2}-2}$

Q.N.7: State and prove relation between Beta and Gamma function.

OR Prove that $\beta(m,n)=\frac{\Gamma m\,\Gamma n}{\Gamma(m+n)}$

Q.N.8: Express $\int_0^1 x^m(1-x^n)^p\,dx$ in terms of Beta function and hence evaluate $\int_0^1 x^5(1-x^3)^{10}\,dx$.

Ans: (i) $\int_0^1 x^m(1-x^n)^p\,dx=\frac{1}{n}\beta\left(\frac{m+1}{n},p+1\right)$ **(ii)** $\frac{1}{396}$.

Q.N.9: Change the order of integration $\int_{y=0}^{a}\int_{x=y}^{a}\frac{x}{x^2+y^2}dydx$ and hence evaluate it. **Ans:** $\frac{a\pi}{4}$.

Q.N.10: (i) Find the volume bounded by cylinder $x^2+y^2=4$, and the planes $y+z=4$ and $z=0$. **Ans** :16π .

(ii) Using triple integral find the volume of the sphere of radius *a* . **Ans:** $V=\frac{4\pi a^3}{3}$.

UNIT – 3 : (Differential equation)

Q.N.11: Solve the Differential equation: $p^2+2py\cot x-y^2=0$

Ans: $[y(1+\cos x)-c_1][y(1-\cos x)-c_2]=0$

Q.N.12: Solve the Differential equation: **(i)** $\frac{d^2y}{dx^2}+4y=x^2+\cos 2x$

Ans: $y=c_1\cos 2x+c_2\sin 2x+\frac{1}{4}\left(x^2-\frac{1}{2}\right)+\frac{x}{4}\sin 2x$

(ii) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x\sin x$ **Ans:** $y=(c_1+xc_2)e^x-e^x\left[x\sin x+2\cos x\right]$

Q.N.13: Solve the Differential equation: $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4y=x^2+2\log x$

Ans: $y=c_1x^4+c_2x^{-1}+c_3e^{-x}-\frac{x^2}{6}-\frac{\log x}{2}+\frac{3}{8}$

Q.N.14: Solve the Differential equation: $\frac{dx}{dt}+y=\sin t, \frac{dy}{dt}+x=\cos t$ given that *x=0* , *y=0* when *t=0* **.**

Ans: $x=e^{t}+e^{-t}, y=\sin t-e^{t}+e^{-t}$

Q.N.15: Solve the Differential equation by method of variation of parameter: Solve: $\frac{d^2y}{dx^2}+4y=4\tan 2x$

Ans: $y=c_1\cos 2x+c_2\sin 2x-\cos 2x\log(\sec 2x+\tan 2x)$

UNIT – 4

MATRICES

Q.N.16: Determine the Rank of the matrix by Normal form:

$$A=\begin{pmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{pmatrix}$$

Ans: Rank=3

Q.N.17: (a) For what value of *k* , the equations : $x+y+z=1, x+2y+4z=k, x+4y+10z=k^2$ have an solution and solve it in each case. **Ans:** The equation have infinite many solutions when *k* = 1, 2

(i) When *k* = 1 : $x=1+2k_1, y=-3k_1, z=k_1$ **(ii)** When *k* = 2 : $x=2k_2, y=1-3k_2, z=k_2$

(b) Investigate the values of λ and μ so that the equations: $2x+3y+5z=9, 7x+3y-2z=8, 2x+3y+\lambda z=\mu$ have **(i)** No solution **(ii)** A unique solution **(iii)** An infinite number of solutions.

Ans: (i) No solution : $\lambda=5, \mu\neq 9$, **(ii)** A unique solution : $\lambda\neq 5, \mu$ have any value.

(iii) An infinite number of solutions : $\lambda=5, \mu=9$

Q.N.18: Find the Eigen values(Characteristic root) and corresponding to Eigen vectors (Characteristic vectors) of the following matrices: A= $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$

Ans: Eigen Values: $\lambda=8,2,2$

Eigen Vectors: $[\{2,-1,1\},\{1,0,-2\},\{0,1,1\}]$

(ii) A= $\begin{pmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{pmatrix}$

Ans: Eigen Values: $\lambda=3,,6,9$

Eigen Vectors: $[\{1,2,2\},\{2,1,-2\},\{2,-2,1\}]$

Q.N.19: Find the Characteristic equation and Verify Cayley Hamilton Theorem for the matrix A= $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$ and Hence compute A^{-1}. **Ans:** $A^{-1} = \frac{1}{2}\begin{pmatrix} -6 & 0 & 4 \\ -2 & 1 & 1 \\ 4 & 0 & -2 \end{pmatrix}$

Q.N.20: (i) Using matrix method, show that the equations: $x+2y=4,\ 3x+3y+2z=1,\ 10y+3z=-2,\ 10y+3z=-2,$ are consistent and hence solve them. **Ans:** $x=2, y=1, z=-4$

(ii) Examine the system of equations: $5x+3y+14z=4, y+2z=1, x-y+2z=0, 2x+y+6z=2$ is consistent. if it is consistent then find the solution. **Ans:** In-consistent

UNIT – 5

ALGEBRA OF LOGIC

Q.N.21: (i) Prove that the following statement is a contradiction:

$$[(p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee q) \wedge (\sim p \vee \sim q)]$$

(ii)) Prove that the following statement is a contradiction:

$$(p \wedge q \Rightarrow r) \Leftrightarrow (p \Rightarrow r) \vee (q \Rightarrow r)$$

Q.N.22: If $(B,+,.,')$ be a Boolean algebra and $a,b \in B$, then prove that (**De-Morgon's law**):

(i) $(a+b)' = a'.b'$ **(ii)** $(a.b)' = a'+b' \quad \forall a,b \in B$

Q.N.23: (i) Express the following function into Conjunctive Normal Form (C.N.F.): $(x+y)(x+y')+(x'+z)$

Ans: $f(x,y,z) = (x+y+z)(x+y+z')(x+y'+z)(x+y'+z')(x'+y+z)(x'+y'+z)$

(ii) Express the following function into Disjunctive Normal Form (D.N.F.): $(x+y+z)(x.y+x'.z')'$

Ans: $f(x,y,z) = xy'z + xy'z' + x'yz + x'y'z$

Q.N.24: Give the definition of Fuzzy logic concept of Fuzzy logic and application of Fuzzy**.**

Q.N.25: `**(a) Define : (i)** Graph **(ii)** Simple Graph **(iii)** Multi-graph **(iv)**Sub-graph **(v)** Degree of a vertex

(b) Prove that the number of vertices of odd degree in a graph is always even.

(c) Show that the maximum number of edges in a simple graph with vertices n is $\frac{n(n-1)}{2}$.